

Exercise 72

A farmer finds there is a linear relationship between the number of bean stalks, n , she plants and the yield, y , each plant produces. When she plants 30 stalks, each plant yields 30 oz of beans. When she plants 34 stalks, each plant produces 28 oz of beans. Find a linear relationship in the form $y = mn + b$ that gives the yield when n stalks are planted.

Solution

The linear equation has the form,

$$y = mn + b.$$

Based on the information, the two points on the line are (30, 30) and (34, 28). The first point says that when the input is 1000, the output is 30.

$$30 = m(30) + b$$

The second point says that when the input is 34, the output is 28.

$$28 = m(34) + b$$

This is a system of two equations for two unknowns that can be solved for.

$$\begin{cases} 30m + b = 30 \\ 34m + b = 28 \end{cases}$$

Subtract the respective sides to eliminate b .

$$30m - 34m = 30 - 28 \quad \rightarrow \quad -4m = 2 \quad \rightarrow \quad m = -\frac{1}{2}$$

Multiply both sides of the first equation by $1/30$, multiply both sides of the second equation by $-1/34$,

$$\begin{cases} m + \frac{b}{30} = 1 \\ -m - \frac{b}{34} = -\frac{14}{17} \end{cases}$$

and then add the respective sides to eliminate m .

$$\frac{b}{30} + \left(-\frac{b}{34}\right) = 1 + \left(-\frac{14}{17}\right) \quad \rightarrow \quad \left(\frac{1}{30} - \frac{1}{34}\right)b = \frac{3}{17} \quad \rightarrow \quad \frac{1}{255}b = \frac{3}{17} \quad \rightarrow \quad b = 45$$

Now that m and b are solved for, the line is known.

$$y = -\frac{1}{2}n + 45$$